



ELSEVIER

8 November 2001

PHYSICS LETTERS B

Physics Letters B 520 (2001) 115–123

www.elsevier.com/locate/npe

c, b quark masses and $f_{D(s)}, f_{B(s)}$ decay constants from pseudoscalar sum rules in full QCD to order α_s^2

Stephan Narison^{a,b}^a *Laboratoire de Physique Mathématique et Théorique, UM2, Place Eugène Bataillon, 34095 Montpellier Cedex 05, France*^b *Center for Academic Excellence on Cosmology and Particle Astrophysics (CosPA), Department of Physics, National Taiwan University, Taipei, 10617 Taiwan, ROC*

Received 30 August 2001; accepted 10 September 2001

Editor: G.F. Giudice

Abstract

The pseudoscalar sum rules of the heavy–light quark systems are used for extracting *simultaneously* the c, b quark masses and the decay constants $f_{D(s)}, f_{B(s)}$ of the $D_{(s)}, B_{(s)}$ mesons. To order α_s^2 , one obtains the running quark masses: $\bar{m}_c(m_c) = (1.10 \pm 0.04)$ GeV, $\bar{m}_b(m_b) = (4.05 \pm 0.06)$ GeV implying $\bar{m}_b(M_Z) = (2.65 \pm 0.05)$ GeV; the perturbative pole masses: $M_c = (1.46 \pm 0.04)$ GeV, $M_b = (4.69 \pm 0.06)$ GeV; and the decay constants: $f_D = (205 \pm 20)$ MeV, $f_B = (203 \pm 23)$ MeV and $f_{D_s} = (235 \pm 24)$ MeV, $f_{B_s} = (236 \pm 30)$ MeV, in the normalization where $f_\pi = 130.56$ MeV. The fitted values of the pole and running masses satisfy quite well their three-loop perturbative relation. The value $f_D \simeq f_B$ confirms earlier findings from the sum rule that the $1/\sqrt{M_P}$ heavy quark symmetry scaling law is affected by large $1/M_P$ corrections.

© 2001 Elsevier Science B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

1. Introduction

One of the most important parameters of the standard model is the quark masses. However, contrary to the leptons, where the physical mass can be identified with the pole of the propagator, the quark masses are difficult to define because of confinement. Some attempts have been made in order to define the heavy quark pole mass within perturbation theory, where it has been shown to be IR-finite [1] and independent of the choice of the regularization and renormalization schemes used [2]. More recently, it has been noticed, in the limit of a large number of flavours, that the resummation of perturbative series can induce a nonper-

turbative term, which can affect the truncated perturbative result, and can, then, limit the accuracy of the pole mass determination (for reviews see, e.g., [3,4]). One may bypass the previous problems, by working, at a given order of perturbative QCD, with the running quark masses, which are treated like coupling constants of the QCD Lagrangian (see, e.g., [5]), and where nonperturbative-like effect is expected to be absent. It is also known that the decay constants $f_{D,B}$ of the pseudoscalar D, B mesons play an essential rôle in the neutral $\bar{D}-D$ and $\bar{B}-B$ oscillations, and in D, B decays, and can directly be measured in the $D^-, B^- \rightarrow l \nu_l$ leptonic decays. In addition, it is important for your knowledge of heavy quark symmetry, how the value of these decay constants deviate from the naïve $1/\sqrt{M_P}$ scaling law, expected to occur when the pseudoscalar meson mass is infinitely

E-mail address: narison@lpm.univ-montp2.fr (S. Narison).

large. A lot of efforts has been furnished in the literature [6] for extracting directly from the data the running masses of the light and heavy quarks and the heavy quark “perturbative” pole masses using the SVZ QCD spectral sum rules (QSSR) [7] (for a complete review, see, e.g., [8]), whilst $f_{D,B}$ come from different forms of the pseudoscalar sum rules [8–15] since the pioneering work of [16]. In this Letter, I shall consider a direct extraction of the running charm and bottom quark masses from pseudoscalar two-point function sum rules where the α_s^2 correction has been recently obtained in [17] and where we shall also use the observed values of the meson masses: $M_{D^-} = 1.869$ GeV and $M_{B^-} = 5.279$ GeV. In the same time, we shall *simultaneously* estimate the decay constants $f_{D(s)}, f_{B(s)}$ of the $D_{(s)}$ and $B_{(s)}$ mesons. The simultaneous extraction of the quark mass and f_P together with the extraction of f_P using the running quark mass has been initiated in our previous work [13,14] to order α_s , and will be improved here to order α_s^2 . All previous works [8–13] with the exception of the recent work in [15] have used, as input, the pole mass value for extracting f_P , where, as we have mentioned previously, the definition of the pole mass might be affected by some nonperturbative contributions [3,4]. Moreover, the extraction of the quark mass value from the pseudoscalar sum rule itself which is an improvement of our previous works [13,14] is not done in all previous works.

2. The QCD spectral sum rules

We shall work with the pseudoscalar two-point correlator:

$$\psi_5(q^2) \equiv i \int d^4x e^{iqx} \langle 0 | T J_q(x) J_q^\dagger(0) | 0 \rangle, \quad (1)$$

built from the heavy–light quark current: $J_d(x) = (m_Q + m_d) \bar{Q}(i\gamma_5)d$, and which has the quantum numbers of the D and B mesons. m_Q is the heavy quark mass, and we shall neglect the d quark mass here. The corresponding Laplace transform sum rules are:

$$\mathcal{L}(\tau) = \int_{t_{\leq}}^{\infty} dt e^{-t\tau} \frac{1}{\pi} \text{Im } \psi_5(t) \quad \text{and}$$

$$\mathcal{R}(\tau) \equiv -\frac{d}{d\tau} \log \mathcal{L}(\tau), \quad (2)$$

where t_{\leq} is the hadronic threshold. The latter sum rule, or its slight modification, is useful, as it is equal to the resonance mass squared, in the simple duality ansatz parametrization of the spectral function:

$$\begin{aligned} \frac{1}{\pi} \text{Im } \psi_5(t) &\simeq 2 f_D^2 M_D^4 \delta(t - M_D^2) \\ &+ \text{“QCD continuum” } \Theta(t - t_c), \end{aligned} \quad (3)$$

where the “QCD continuum” comes from the discontinuity of the QCD diagrams, which is expected to give a good smearing of the different radial excitations.¹ The decay constant f_D is analogous to $f_\pi = 92.32$ MeV. However, in order to avoid some confusion, and for a more direct comparison with the lattice results, we shall abandon our favorite normalization, and adopt in the rest of the Letter, the one:

$$f_D \equiv \sqrt{2} f_D, \quad (4)$$

a normalization where $f_\pi = 130.56$ MeV, and which the different experimental groups have also adopted; t_c is the QCD continuum threshold, which is, like the sum rule variable τ , an (a priori) arbitrary parameter. In this Letter, we shall impose the t_c and τ stability criteria for extracting our optimal results.² The QCD expression of the correlator is well-known to two-loop accuracy (see, e.g., [8] and the explicit expressions given in [9]), in terms of the perturbative pole mass M_Q , and including the nonperturbative condensates of dimensions less than or equal to six.³ The sum rule reads:

$$\mathcal{L}(\tau) = M_Q^2 \left\{ \int_{M_Q^2}^{\infty} dt e^{-t\tau} \frac{1}{8\pi^2} \right.$$

¹ At the optimization scale, its effect is negligible, such that a more involved parametrization is not necessary.

² The corresponding t_c value very roughly indicates the position of the next radial excitations.

³ A different expression of the coefficient of the quark–gluon mixed condensate is given in [18]. This change affects only slightly the result. We shall include the negligible contribution from the dimension six four-quark condensates. Notice that there is some discrepancy on the value of the four-quark coefficient in the literature.

$$\times \left[3t(1-x)^2 \left(1 + \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) f(x) \right) + \left(\frac{\alpha_s}{\pi} \right)^2 R2s \right] + [pC_4\langle O_4 \rangle + C_6\langle O_6 \rangle \tau] e^{-M_Q^2 \tau} \Bigg\}, \quad (5)$$

where $R2s$ is the new α_s^2 -term obtained semi-analytically in [17] and is available as a MATHEMATICA package program Rvs.m. The other terms are:

$$\begin{aligned} x &\equiv M_Q^2/t, \\ f(x) &= \frac{9}{4} + 2\text{Li}_2(x) + \log x \log(1-x) \\ &\quad - \frac{3}{2} \log(1/x - 1) - \log(1-x) \\ &\quad + x \log(1/x - 1) - (x/(1-x)) \log x, \\ C_4\langle O_4 \rangle &= -M_Q \langle \bar{d}d \rangle + \langle \alpha_s G^2 \rangle / 12\pi, \\ C_6\langle O_6 \rangle &= \frac{M_Q^3 \tau}{2} \left(1 - \frac{1}{2} M_Q^2 \tau \right) g \left\langle \bar{d} \sigma_{\mu\nu} \frac{\lambda_a}{2} G_a^{\mu\nu} d \right\rangle \\ &\quad - \left(\frac{8\pi}{27} \right) \left(2 - \frac{M_Q^2 \tau}{2} - \frac{M_Q^4 \tau^2}{6} \right) \\ &\quad \times \rho \alpha_s \langle \bar{\psi} \psi \rangle^2. \end{aligned} \quad (6)$$

The previous sum rules can be expressed in terms of the running mass $\bar{m}_Q(v)$,⁴ through the perturbative three-loop relation [1,2,19]:

$$\begin{aligned} M_{\text{pole}} &= \bar{m}(p^2) \left[1 + \left(\frac{4}{3} + \ln \frac{p^2}{M^2} \right) \left(\frac{\bar{\alpha}_s}{\pi} \right) \right. \\ &\quad + \left[K_Q + \left(\frac{221}{24} - \frac{13}{36} n \right) \ln \frac{p^2}{M^2} \right. \\ &\quad \left. \left. + \left(\frac{15}{8} - \frac{n}{12} \right) \ln^2 \frac{p^2}{M^2} \right] \left(\frac{\bar{\alpha}_s}{\pi} \right)^2 \right], \end{aligned} \quad (7)$$

⁴ It is clear that, for the nonperturbative terms which are known to leading order of perturbation theory, one can use either the running or the pole mass. However, we shall see that this distinction does not affect notably the present result.

where, in the RHS, $M_{\text{pole}} \equiv M$ is the pole mass and:

$$K_Q = 17.1514 - 1.04137n + \frac{4}{3} \sum_{i \neq Q} \Delta \left(r \equiv \frac{m_i}{M_Q} \right). \quad (8)$$

For $0 \leq r \leq 1$, $\Delta(r)$ can be approximated, within an accuracy of 1% by:

$$\Delta(r) \simeq \frac{\pi^2}{8} r - 0.597r^2 + 0.230r^3. \quad (9)$$

Throughout this Letter we shall use the values of the parameters [8,20] given in Table 1.

We have used for the mixed condensate the parametrization:

$$g \left\langle \bar{d} \sigma_{\mu\nu} \frac{\lambda_a}{2} G_a^{\mu\nu} d \right\rangle = M_0^2 \langle \bar{d}d \rangle, \quad (10)$$

and we deduce the value of the QCD scale Λ from the value of $\alpha_s(M_Z) = (0.1184 \pm 0.031)$ given in [6,21].

3. The D -meson channel

We study in Fig. 1, the prediction of f_D from the Laplace sum rules \mathcal{L} and the one of M_D from ratio of moments \mathcal{R} for given value of the running charm quark mass $\bar{m}_c(m_c)$. The influences of the choice of the continuum threshold t_c and of the sum rule scale τ are shown in details. Our optimal results correspond to the case where both stability in τ and in t_c are reached. However, for a more conservative estimate of the errors we allow deviations from the stability points, and we take:

$$\begin{aligned} t_c &\simeq (6 \sim 9.5) \text{ GeV}^2, \\ \tau &\simeq (1.2 \pm 0.2) \text{ GeV}^{-2}, \end{aligned} \quad (11)$$

and where the lowest value of t_c corresponds to the beginning of the τ -stability region. Values outside the above ranges are not consistent with the stability criteria. One can inspect that the dominant nonperturbative contribution is due to the dimension-four $M_c \langle \bar{d}d \rangle$ light quark condensate, and test that the OPE is not broken by high dimension condensates at the optimization scale. However, the perturbative radiative corrections converge slowly, as the value of f_D increases by 12% after the inclusion of the α_s correction and the sum of the lowest order plus α_s -correction increases by 21% after the inclusion of the α_s^2 term, indicating

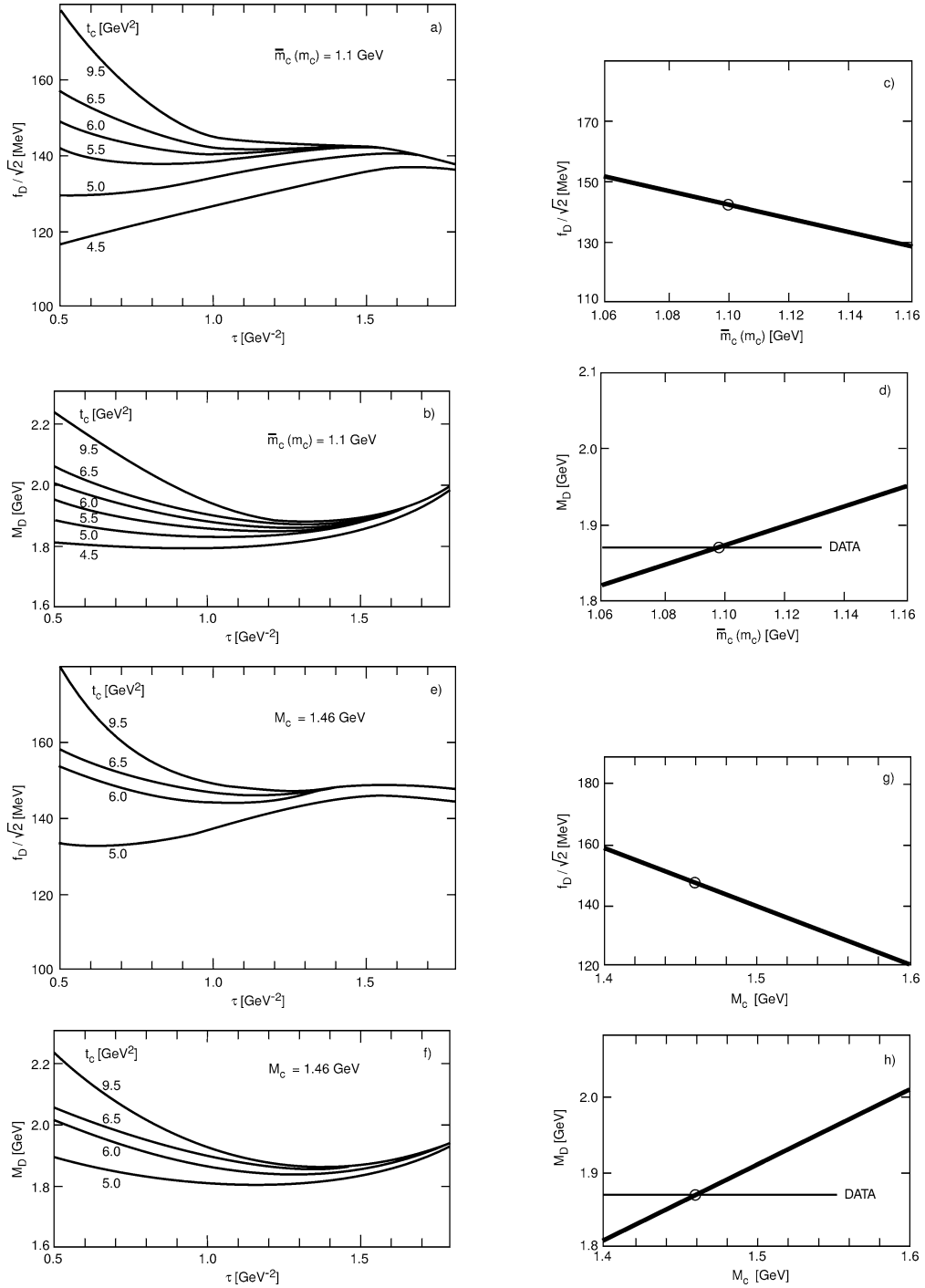


Fig. 1. Laplace sum rule analysis of the decay constant f_D , the running mass $\bar{m}_c(m_c)$ and the pole mass M_c : (a) f_D versus the sum rule scale τ at given $\bar{m}_c(m_c)$ and for different values of the continuum threshold t_c ; (b) the same as (a) but for M_D ; (c) and (d) effects of $\bar{m}_c(m_c)$ on M_D and f_D . The circle is the solution given by the data on M_D ; (e) to (h) the same as (a) to (f) but for the pole mass.

Table 1

Different sources of errors in the estimates of the decay constants (in MeV) and quark masses (in GeV). We have exagerately enlarged the error bars of different input in order to have conservative errors

Sources	$ \Delta f_D _{\bar{m}_c}$	$ \Delta f_D _{M_c}$	$ \Delta \bar{m}_c $	$ \Delta M_c $	$ \Delta f_B _{\bar{m}_b}$	$ \Delta f_B _{M_b}$	$ \Delta \bar{m}_b $	$ \Delta M_b $
$\Lambda_4 = (325 \pm 43) \text{ MeV}$	7.4	6.2	0.03	0.03	–	–	–	–
$\Lambda_5 = (225 \pm 30) \text{ MeV}$	–	–	–	–	3.6	3.0	0.02	0.02
$v = M_{c,b} \pm 1/2(M_{c,b} - \bar{m}_{c,b})$	9.3	–	–	–	14.0	–	–	–
Geom. estimate of α_s^3 -term	7.9	8.2	–	–	1.7	2.0	–	–
$\tau^D = (1.2 \pm 0.2) \text{ GeV}^{-2}$	1.1	1.1	0.01	0.01	–	–	–	–
$\tau^B = (0.35 \pm 0.05) \text{ GeV}^{-2}$	–	–	–	–	8.2	5.7	0.02	0.02
$6.0 \leq t_c^D [\text{GeV}^2] \leq 9.5$	2.1	2.8	0.01	0.01	–	–	–	–
$36 \leq t_c^B [\text{GeV}^2] \leq 50$	–	–	–	–	2.4	2.8	0.05	0.03
$\langle \bar{d}d \rangle^{1/3}(1 \text{ GeV}) = -(229 \pm 18) \text{ MeV}$	8.8	8.9	0.01	0.01	5.2	7.1	–	–
$\langle \alpha_s G^2 \rangle = (0.07 \pm 0.03) \text{ GeV}^2$	1.8	1.1	0.005	0.005	0.9	1.0	–	–
$M_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$	0.7	0.7	0.005	0.01	2.2	2.1	0.03	0.04
$\alpha_s \langle \bar{\psi}\psi \rangle^2 = (5.8 \pm 2.4) \times 10^{-4} \text{ GeV}^6$	0.2	0.3	–	–	0.6	1.0	0.01	0.01
From our estimate of $m_{c,b}$ or $M_{c,b}$	10.9	9.6	–	–	15	13	–	–
Total	20	17	0.04	0.04	23	17	0.06	0.06

that the total amount of corrections of 21% is still a reasonable correction despite the slow convergence of the perturbative series, which might be improved using, like in [10], a resummed series. However, as the radiative corrections are both positive, we expect that this slow convergence will not affect in a sensible way the final estimate. In order to improve the perturbative contributions, we have estimated the possible contribution of the α_s^3 -term by assuming that its coefficient is the geometric sum of the α_s and α_s^2 contributions. This effect is shown in Table 1, which is has a quite reasonable value. A more precise answer on the higher order perturbative contribution needs an evaluation of the α_s^3 term which we hope to be available in the near future. In doing the analysis, one can also notice that the relative size of the perturbative corrections is smaller in the physical observable f_D (perturbative + nonperturbative) than in the perturbative graph alone. This is due to fact that the rôle of the $\langle \bar{\psi}\psi \rangle$ condensate is important at the optimization scale, which then decreases the relative weight of the perturbative radiative corrections in the OPE. The estimate of M_D from the ratio of moments \mathcal{R} is less affected by radiative corrections, which tend to cancel each others due

to the form of the sum rule. The behaviour of the optimized values of f_D and M_D versus different values of $\bar{m}_c(m_c)$ is given in Figs. 1(a) and (b). One can explicitly see in Figs. 1(c) and (d) that both M_D and f_D are very sensitive to the change of $\bar{m}_c(m_c)$. This feature allows to have a good determination of the quark mass and then of f_D , once the experimental value of M_D is used. Adding quadratically the different errors given in Table 1, we deduce the final estimate:

$$\begin{aligned} \bar{m}_c(m_c) &= (1.10 \pm 0.04) \text{ GeV}, \\ f_D &= (201 \pm 20) \text{ MeV}, \end{aligned} \quad (12)$$

where as mentioned previously, we have used the normalization $f_\pi = 130.56 \text{ MeV}$. These optimal values correspond to $t_c = 6.5 \text{ GeV}^2$ and $\tau = 1.2 \text{ GeV}^{-2}$. The value of t_c roughly corresponds to a radial excitation with a mass-splitting relative to the ground state mass of about M_ρ which is phenomenologically acceptable. A similar analysis shown in Figs. 1(e)–(h) is done for the pole mass. The discussions presented previously apply also here, including the one of the radiative corrections. We quote the final result:

$$M_c = (1.47 \pm 0.04) \text{ GeV},$$

$$f_D = (208 \pm 17) \text{ MeV}, \quad (13)$$

where the error is slightly smaller here due to the absence of the subtraction scale uncertainties. For f_D , we consider as a final estimate the mean value of the two predictions and taking the largest errors:

$$f_D = (204 \pm 20) \text{ MeV}. \quad (14)$$

Using our previous estimate of $f_{D_s}/f_D = 1.15 \pm 0.04$ [13], we can also deduce:

$$f_{D_s} = (235 \pm 24) \text{ MeV}. \quad (15)$$

One can immediately compare the present predictions with the one obtained to order α_s using the same procedure [14]:

$$\bar{m}_c(m_c) = (1.08 \pm 0.11) \text{ GeV},$$

$$f_D = (201 \pm 15) \text{ MeV}, \quad (16)$$

where one can notice a good agreement between the α_s and α_s^2 results, though the error in [14] is smaller as the effect of the subtraction point ν has not been taken into account. However, the agreement seems paradoxical in view of the fact that radiative corrections tend to increase the value of f_D compared to the lower orders result. The different truncations of the expression of α_s and of the relation between the pole and running mass also affect the absolute value of f_D , which tend to compensate the increase due to the radiative corrections of the correlator. Therefore, a naïve comparison becomes misleading. As one can see in Table 1, the main source of errors is due to the variations of the quark mass and to a lesser extent to the ones of ν , Λ and $\langle \bar{\psi}\psi \rangle$. The effect of t_c on the result is relatively small from the value t_c larger than 6 GeV^2 , where one starts to have a τ stability. This result for the mass is also in agreement with the one from $M_{J/\psi}$ [6,22]:

$$\bar{m}_c(m_c) = (1.23^{+0.04}_{-0.05}) \text{ GeV}, \quad (17)$$

but lower than the one from [23] using nonrelativistic Balmer formula for the $\bar{c}c$ bound state. One can cross-check that the two values of $\bar{m}_c(m_c)$ and M_c give the ratio:

$$M_c/\bar{m}_c(m_c) \simeq 1.33, \quad (18)$$

which satisfies quite well the three-loop perturbative relation $M_c/\bar{m}_c(m_c) = 1.33$ obtained from the previous Eq. (7). This could be a nontrivial result if

one has in mind that the quark pole mass definition can be affected by nonperturbative corrections not present in the standard SVZ-OPE. In particular, it may signal that $1/q^2$ correction of the type discussed in [3,24], if present, will only affect weakly the standard SVZ-phenomenology as observed explicitly in the light quark, gluonia and hybrid channels [24]. Recent results to order α_s^2 [12], using the analogous sum rule and using as input the pole mass value, gives $f_D = (195 \pm 20) \text{ MeV}$. The result is slightly lower than the result given here, though in agreement within the errors. We understand this slight difference as due to the lower value of the QCD continuum threshold used there, which corresponds to $t_c \approx 5.6 \text{ GeV}^2$ if one uses our pole mass value $M_c = 1.47 \text{ GeV}$. As shown in Fig. 1(e), this t_c value is on the boarder of the τ stability region. It also indicates that the error introduced by the choice of t_c could have been underestimated in this result. Fig. 1(g) also indicates that f_D is quite sensitive in full QCD to the change of the M_c value, while the weaker dependence obtained in [12] is also related to the values of t_c used there. Quenched and unquenched lattice results for f_D are compiled in [25]. The quenched results range from (192 ± 18) to $(221 \pm 17) \text{ MeV}$, in fair agreement with our results within the errors. The two available unquenched results [26,27] lead to the average:

$$\begin{aligned} f_D^{\text{lat}} &= (220 \pm 20) \text{ MeV}, \\ f_{D_s}^{\text{lat}} &= (254 \pm 29) \text{ MeV}, \end{aligned} \quad (19)$$

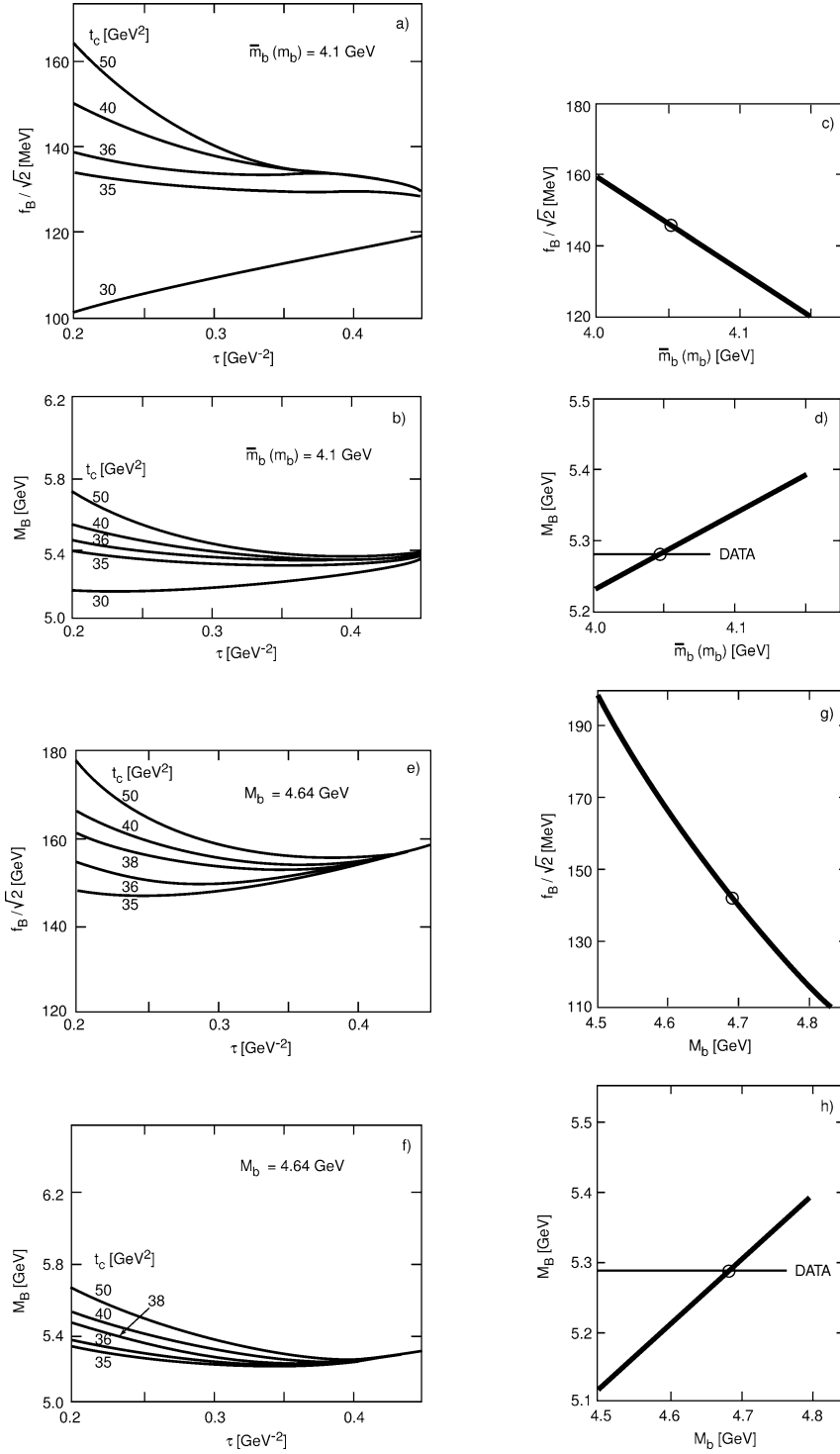
which is again in agreement within the errors with our previous estimate. Finally, one can also compare the value of f_{D_s} with the experimental value [29]:

$$f_{D_s}^{\text{exp}} = (286 \pm 60) \text{ MeV}, \quad (20)$$

which agrees within 1σ with our prediction. Improvements of our predictions for f_{D_s} need an estimate of the ratio f_{D_s}/f_D to order α_s^2 for the m_s and $\langle \bar{s}s \rangle$ corrections.

4. The B -meson channel

We extend the previous analysis to the case of the B -meson, which again is an update of our previous work in [8,9,14]. The analysis is still similar to the one done in the D -channel, and is summarized in Fig. 2

Fig. 2. The same as Fig. 1 but for the b -quark and B -meson.

and in Table 1. Using the running b -quark mass, as a free parameter, we obtain at the optimization scale $\tau = 0.375 \text{ GeV}^{-2}$ and $t_c = 38 \text{ GeV}^2$:

$$\begin{aligned}\bar{m}_b(m_b) &= (4.05 \pm 0.06) \text{ GeV}, \\ f_B &= (205 \pm 23) \text{ MeV},\end{aligned}\quad (21)$$

while using the pole mass as a free parameter, we get:

$$\begin{aligned}M_b &= (4.69 \pm 0.06) \text{ GeV}, \\ f_B &= (200 \pm 17) \text{ MeV},\end{aligned}\quad (22)$$

from which we deduce the average:

$$f_B = (203 \pm 23) \text{ MeV}, \quad (23)$$

where we have taken the largest errors. One can again cross-check that the two values of $\bar{m}_b(m_b)$ and M_b lead to

$$M_b / \bar{m}_b(m_b) = 1.16, \quad (24)$$

to be compared with 1.15 from the three-loop perturbative relation in Eq. (7), and might indirectly indicate the smallness of the $1/q^2$ correction if any. Our result of \bar{m}_b can be compared with our previous estimate from \mathcal{Y} -systems [22]:

$$\bar{m}_b(m_b) = (4.23 \pm 0.05) \text{ GeV}, \quad (25)$$

and with similar values from recent estimates [6, 30]. Our value of the perturbative pole mass is in agreement within the errors with the one in [6,22,30] but is again lower than the one in [23]. Our value of f_B is in fair agreement with the recent results $f_B = (206 \pm 20) \text{ MeV}$ obtained in [12] using HQET sum rules and the one $f_B = (197 \pm 23) \text{ MeV}$ obtained in [15] using the Laplace sum rule like in this work. The slight difference is due to the different appreciations of the continuum threshold t_c and τ stability regions in each papers. More specifically, errors related to the choice of t_c at their choice of lower τ -values appear to be underestimated in these works. At such a choice of low τ -values, the ground state contribution to the sum rule is smaller than in the present analysis. Unquenched lattice results [27,28] give the mean value:

$$\begin{aligned}f_B^{\text{lat}} &= (198 \pm 37) \text{ MeV}, \\ f_{B_s}^{\text{lat}} / f_B^{\text{lat}} &= 1.17 \pm 0.03,\end{aligned}\quad (26)$$

where the largest error has been taken. These values agree with our previous determination in Eq. (23) and with our earlier estimate [13]:

$$f_{B_s} / f_B = 1.16 \pm 0.05, \quad (27)$$

which has been confirmed from the recent analysis of [15]. Combining this $SU(3)$ breaking ratio with our estimate of f_B , one obtains:

$$f_{B_s} = (236 \pm 30) \text{ MeV}. \quad (28)$$

The extension of the previous analysis to the D^* and B^* channels is in progress.

5. Summary

We have updated our previous estimate of the quark masses and decay constants in [8,9,14] using the recent expression [17] of α_s^2 corrections for the heavy–light pseudoscalar correlators. Our results for the masses in Eqs. (12), (13), (21), (22) and for the decay constants in Eqs. (14), (15), (23), (28) confirm previous estimates to two-loops. The results for the running masses are:

$$\begin{aligned}\bar{m}_c(m_c) &= (1.10 \pm 0.04) \text{ GeV}, \\ \bar{m}_b(m_b) &= (4.23 \pm 0.05) \text{ GeV},\end{aligned}\quad \text{Eqs. (12), (21),}$$

where $\bar{m}_b(m_b)$ runned until the Z -mass implies to order α_s^2 :⁵

$$\bar{m}_b(M_b) = (2.65 \pm 0.05) \text{ GeV}.$$

The pole masses are:

$$\begin{aligned}M_c &= (1.47 \pm 0.04) \text{ GeV}, \\ M_b &= (4.69 \pm 0.06) \text{ GeV},\end{aligned}\quad \text{Eqs. (13), (22).}$$

The decay constants are:

$$\begin{aligned}f_D &= (204 \pm 20) \text{ MeV}, \\ f_B &= (203 \pm 23) \text{ MeV},\end{aligned}\quad \text{Eqs. (14), (23).}$$

Using our previous $SU(3)$ breaking prediction on f_{P_s}/f_P [13], one also deduces:

$$\begin{aligned}f_{D_s} &= (235 \pm 24) \text{ MeV}, \\ f_{B_s} &= (236 \pm 30) \text{ MeV},\end{aligned}\quad \text{Eqs. (15), (28).}$$

⁵ Motivated by a remark of Gordon Kane.

The three-loop corrections tend to push the values of the decay constants to higher values restoring the slight discrepancy between the sum rules and recent unquenched lattice values. The resulting equality $f_D \simeq f_B$ confirm earlier findings from the sum rule [9] indicating large corrections to the $1/\sqrt{M_P}$ heavy quark symmetry scaling law. Values of the quark masses obtained from the pseudoscalar sum rules are in agreement with the one from the quarkonia sum rules [6,22,30], but lower than the ones obtained in [23] from nonrelativistic Balmer formulae. The fitted values of the running and perturbative pole masses satisfy quite well their three-loop perturbative relation, which may indicate that $1/q^2$ -like terms [3,24] have negligible effect in this channel like in the case of the light quark systems.

Acknowledgements

It is a pleasure to thank W.-Y. Pauchy Hwang for the hospitality at CosPA-NTU (Taipei), where part of this work has been done, Abdesslam Arhrib for his help in running the three-loop package Rvs.m with his newer MATHEMATICA version, Matthias Steinhauser on the use of Rvs.m, Arifa Ali Khan for commenting the CP-PAC results and Valya Zakharov for some communications.

References

- [1] R. Tarrach, Nucl. Phys. B 183 (1981) 384.
- [2] S. Narison, Phys. Lett. B 197 (1987) 405;
S. Narison, Phys. Lett. B 216 (1989) 191.
- [3] F.V. Gubarev, M.I. Polikarpov, V.I. Zakharov, Nucl. Phys. (Proc. Suppl.) B 86 (2000) 457;
V.I. Zakharov, Nucl. Phys. (Proc. Suppl.) B 74 (1999) 392;
R. Akhoury, V.I. Zakharov, Nucl. Phys. (Proc. Suppl.) A 54 (1997) 217.
- [4] M. Beneke, Phys. Rep. 317 (1999) 1.
- [5] E.G. Floratos, S. Narison, E. de Rafael, Nucl. Phys. B 155 (1979) 115.
- [6] PDG 2000, D.E. Groom et al., Eur. Phys. J. C 15 (2000) 1.
- [7] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385;
M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 448.
- [8] For a review and references to original works, see, e.g., S. Narison, in: QCD Spectral Sum Rules, Lecture Notes in Physics, Vol. 26, World Scientific, 1989;
S. Narison, Acta Phys. Pol. 26 (1995) 687;
S. Narison, QCD as a theory of hadrons, to be published by Cambridge Univ. Press.
- [9] S. Narison, Phys. Lett. B 198 (1987) 104;
S. Narison, Phys. Lett. B 210 (1988) 238, where references to earlier works can be found.
- [10] E. Bagan et al., Phys. Lett. B 278 (1992) 457;
M. Neubert, Phys. Rev. D 45 (1992) 2451.
- [11] For reviews, see, e.g., C.A. Dominguez, in: J. Kirkby, R. Kirkby (Eds.), 3rd Workshop on Tau-Charm Factory Marbella-Spain, 1993;
S. Narison, in: J. Kirkby, R. Kirkby (Eds.), 3rd Workshop on Tau-Charm Factory, Marbella-Spain, 1993;
BABAR Collaboration, P.F. Harrison, H.R. Quinn, Slac-R-0504, 1997;
P. Colangelo, A. Khodjamirian, hep-ph/0010175.
- [12] A.A. Penin, M. Steinhauser, hep-ph/0108811, and private communications.
- [13] S. Narison, Phys. Lett. B 322 (1994) 247.
- [14] S. Narison, Nucl. Phys. (Proc. Suppl.) B 74 (1999) 304.
- [15] M. Jamin, B.O. Lange, hep-ph/0108135.
- [16] V.A. Novikov et al., Neutrino 78, Purdue Lafayette, 1978.
- [17] K.G. Chetyrkin, M. Steinhauser, Phys. Lett. B 502 (2001) 104, hep-ph/0108017.
- [18] S. Generalis, J. Phys. Nucl. Part. Phys. G 16 (1990) 367.
- [19] N. Gray, D.J. Broadhurst, K. Schilcher, Z. Phys. C 48 (1990) 673.
- [20] S. Narison, Phys. Lett. B 361 (1995) 121;
S. Narison, Phys. Lett. B 387 (1996) 162.
- [21] S. Bethke, Nucl. Phys. (Proc. Suppl.) B, A 54 (1997), hep-ex/0004021;
I. Hinchliffe, Meeting of the American Physical Society, Albuquerque, 1994;
M. Schmelling, ICHEP '96, Warsaw, 1996.
- [22] S. Narison, Phys. Lett. B 341 (1994) 73;
S. Narison, Phys. Lett. B 216 (1989) 191.
- [23] A. Pineda, F.J. Yndurain, Phys. Rev. D 61 (2000) 077505.
- [24] K. Chetyrkin, S. Narison, V.I. Zakharov, Nucl. Phys. B 550 (1999) 353;
K. Chetyrkin, S. Narison, Phys. Lett. B 485 (2000) 145.
- [25] M. Ciuchini et al., hep-ph/001308.
- [26] CP-PAC Collaboration, A. Ali Khan et al., hep-lat/0010009.
- [27] MILC Collaboration, C. Bernard et al., hep-lat/0011029.
- [28] CP-PAC Collaboration, A. Ali Khan et al., hep-lat/0103020.
- [29] OPAL Collaboration, G. Abbiendi et al., hep-ex/0103012.
- [30] M. Jamin, A. Pich, Nucl. Phys. (Proc. Suppl.) B 74 (1999) 30;
A.A. Penin, A.A. Pivovarov, Nucl. Phys. B 549 (1999) 217;
M. Beneke, A. Signer, Phys. Lett. B 471 (1999) 233;
A.H. Hoang, Phys. Rev. D 61 (2000) 034005;
V. Gimenez et al., JHEP 03 (2000) 018.